Electroweak Gauge-Boson and Higgs Production at Small q_T
Infrared Safety from the Collinear Anomaly

Thomas Becher[1], *Matthias Neubert*[2], *Daniel Wilhelm*[2,3]

[1]Institut für Theoretische Physik, Universität Bern, Switzerland
[2]Institut für Physik (THEP), Johannes Gutenberg-Universität Mainz, Germany
[3]Presenter

DOI: `http://dx.doi.org/10.3204/DESY-PROC-2012-02/314`

We study the differential cross sections for electroweak gauge-boson and Higgs production at small and very small transverse-momentum q_T. Large logarithms are resummed using soft-collinear effective theory. The collinear anomaly generates a non-perturbative scale q_*, which protects the processes from receiving large long-distance hadronic contributions. A numerical comparison of our predictions with data on the transverse-momentum distribution in Z-boson production at the Tevatron and LHC is given.

1 Introduction

In multi-scale processes with several disparate scales, large logarithms of scale ratios spoil the perturbative expansion of fixed-order calculations. To obtain a reliable theory prediction one has to resum these logarithms to all orders. Traditional resummation approaches often suffer from cut-off effects originating from the regularization of Landau-pole singularities. One way to avoid such complications is to factorize the observable via an appropriate effective field theory (EFT), describing the low-energy degrees of freedom using effective quark and gluon fields and resum large logarithms using renormalization group (RG) techniques.

We studied the differential cross section for Drell-Yan like gauge-boson production at hadron colliders [1][2], in the regime where the transverse-momentum q_T of the produced boson (or lepton pair) is small compared to its invariant mass M. We considered in detail real production of W- and Z-bosons and the decay of massive ($M \gg q_T$) virtual photons into lepton pairs. The region of small q_T is of great phenomenological importance, since it has the largest cross section and is used e.g. to extract the W-boson mass and width. Pioneering work in this field was done in [3], but this is the first time the resummation was done directly in momentum space. The extension of the used formalism to Higgs-boson production via gluon-fusion can be achieved in a straightforward manner. Here the region of small q_T is important because one usually vetoes hard jets in order to enhance the signal over background ratio.

2 Factorization in soft-collinear effective theory

In contrast to the naive expectation, the underlying scale of the needed EFT for Drell-Yan like processes is not only the small scale q_T. The appearance of Sudakov double logarithms [4] at 1-loop-order automatically generates a new soft scale w, correlated to the hard and collinear scales M and q_T, which becomes obvious by decomposing such a logarithm:

$$\ln^2 \tfrac{M^2}{q_T^2} = \tfrac{1}{2}\left[\ln^2 \tfrac{M^2}{\mu^2} - \ln^2 \tfrac{q_T^2}{\mu^2} - \ln^2 \tfrac{q_T^2}{\mu^2} + \ln^2 \tfrac{w^2}{\mu^2}\right] \ , \quad w = \tfrac{q_T^2}{M} \ .$$

The appropriate EFT describing these degrees of freedom is the soft-collinear effective theory (SCET). In general SCET contains any number of “collinear" fields describing high-energetic lightlike particles (or jets) and soft fields, which mediate the only interactions between the different collinear modes. In our case there are two collinear particles, the two colliding hadrons, whose momenta are characterized best in lightcone coordinates. Therefore we introduce two lightlike reference vectors n and $\bar{n}$ along the beam axis. Now every 4-vector k can be decomposed into its collinear k_+, anticollinear k_- and transverse component $k_\perp (k_T^2 = -k_\perp^2)$, by projecting it onto n and $\bar{n}$. The values of interest are the scalings of momenta in these components and their virtuality $\sqrt{k^2}$, described by the small expansion parameter $\lambda = \frac{q_T}{M}$ (Table 1). To receive the SCET Lagrangian one integrates out all hard modes, defined by their virtuality, in our case the produced boson. After a field redefinition even the soft modes decouple from the two collinear modes and one can match hadronic matrix-elements onto operators in SCET, which leads directly to a factorized cross section:

Mode	$\frac{1}{M}(k_+, k_-, k_T)$	Virtuality
Hard	$q \sim (\,1\,,\,1\,,\lambda\,)$	M
Collinear	$p \sim (\,1\,,\lambda^2,\lambda\,)$	$M\lambda \ \sim q_T$
Anticollinear	$\bar{p} \sim (\lambda^2,\,1\,,\lambda\,)$	$M\lambda \ \sim q_T$
Soft	$k \sim (\lambda^2,\lambda^2,\lambda^2)$	$M\lambda^2 \sim w$

Table 1: Scaling of involved particles.

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}q_T \mathrm{d}y} = A \cdot H \cdot \sum_{ij} Q_{ij} \cdot \tfrac{1}{4\pi}\int \mathrm{d}^2\vec{x}_\perp \mathrm{e}^{-i\vec{q}_\perp \vec{x}_\perp} \cdot W \cdot \mathcal{B}_{i/P_1}\mathcal{B}_{j/P_2} + \mathcal{O}\left(\lambda^2\right) \ . \tag{1}$$

The kinematic prefactor A is not affected by the matching and can contain the leptonic part describing the decay of the boson. H denotes the hard function depending only on the hard scale M and containing the Wilson coefficients. The hadronic matrix element factorizes in a soft function W and two collinear functions $\mathcal{B}$, which are summed over contributing partons with effective charges Q. The Wilson coefficients and thus the hard functions are known at least to two-loop order. The soft modes do not contribute, since the soft function is equal to $W = 1 + \mathcal{O}(\lambda)$ for all orders. The collinear functions $\mathcal{B}$ are generalized, x_T dependent parton-distribution-functions (PDF), and can be matched at the partonic level onto ordinary PDFs. So the cross section seems to be calculable straightforwardly, but there are more subtle obstacles due to the collinear anomaly, which will be discussed in the next section.

3 Collinear anomaly and infrared safety

Two problems appear in the factorized formula (1). The first one is related to the renormalization invariance. As a physical observable, the cross section should be invariant under the change of μ, thus its derivative with respect to μ has to give zero. The derivative of the hard function is known and leads to terms proportional to logarithms of the hard scale M, which should be compensated by the derivatives of the other factors. But, in the absence of the soft contribution, there is no term depending on M, thus the cross section seems to be not scale invariant. The second problem appears in the matching of the generalized PDFs $\mathcal{B}$ onto ordinary PDFs, because they lead to integrals which cannot be regularized in dimensional regularization.

Both problems originate directly from the collinear anomaly (CA), a real quantum anomaly in SCET, in the sense that a symmetry is broken by quantum corrections. At LO the two collinear Lagrangians are invariant under the so-called rescaling transformation, which is given by multiplying all (anti-)collinear momenta by a real factor $(\bar{a})a$. But at higher orders, this symmetry is broken and restricted to $\bar{a} \cdot a \equiv 1$, thus the product $p \cdot \bar{p}$ proportional to M^2 is a new invariant in the EFT. Knowing the origin of the divergences, one can analytically regulate the one-loop diagrams in the matching procedure. A gauge invariant way is to change the phasespace integral according to [5]. One collinear function $\mathcal{B}$ alone is not well-defined, only their product is regulator independent and an anomalous dependency on the hard scale factors out, which ensures the RG invariance of the whole cross section:

$$\mathcal{B}_{i/P_1} \cdot \mathcal{B}_{j/P_2} \to \left(x_T^2 M^2\right)^{-F_{ij}\left(x_T^2,\mu\right)} \cdot B_{i/P_1}\left(x_T^2,\mu\right) \cdot B_{j/P_2}\left(x_T^2,\mu\right)$$

Now the two problems are solved, the only remaining question is the choice of the renormalization scale μ. The idea is of course to choose μ such that the not resummed logarithms remain small. The collinear functions B depend on μ via $L_\perp = \ln\left(x_T^2\mu^2\right)$, so the choice of the renormalization scale similar to the reciprocal transverse displacement $\mu \sim \frac{1}{x_T}$ would lead to small logarithms. But as an integration variable of the Fourier transformation, x_T is not an underlying scale of the process. The next idea could be to choose μ similar to the conjugate variable of x_T: $\mu \sim q_T$. To verify this choice one has to evaluate the Fourier integral. At LO this leads to the analytically solvable integral K_0 (2). At higher orders the only difference is the appearance of powers of $L_\perp$, so these integrals K_n can be written as derivatives of K_0 with respect to η (3), which makes obvious that the choice $\mu \sim q_T$ leads to small logarithms:

$$K_0 \sim \int \mathrm{d}^2\vec{x}_\perp \mathrm{e}^{-i\vec{q}_\perp\vec{x}_\perp} \cdot \mathrm{e}^{-\eta L_\perp} \sim \left(\frac{q_T^2}{\mu^2}\right)^\eta \frac{\Gamma\left(1-\eta\right)}{\Gamma\left(\eta\right)} \tag{2}$$

$$K_n = \left(-\partial_\eta\right)^n K_0 \sim \ln^n\left(\frac{q_T^2}{\mu^2}\right) \qquad\qquad \eta = \frac{\alpha_s}{4\pi}\Gamma_0 \ln\frac{M^2}{\mu^2} \tag{3}$$

The parameter η in the exponent of K_0 represents the M dependence originating from the CA. Choosing $\mu \sim q_T$, η is a small number at high q_T and increases as one lowers q_T. The solution of K_0 introduces a new scale q_*, where η becomes equal to 1 and K_0 diverges (Gamma function in 2). For the Z-boson this scale is around $q_*^Z \approx 1.8$ GeV and for the Higgs $q_*^H \approx 7.7$ GeV, so it lies in the perturbative domain $q_* > \Lambda_{NP}$.

To lower q_T beyond q_*, one has to dismiss the demand of small logarithms $\alpha_s L_\perp \sim \mathcal{O}\left(\alpha_s\right)$, so even at LO one has to take more terms in the exponent of K_0 into account. The next term is quadratic in $L_\perp$ and negative, so the integral becomes a Gaussian. Considering this integral at $\mu \sim q_*$ the Gaussian regulates it with an expectation value of $L_\perp \sim \mathcal{O}\left(1\right)$ and a standard deviation of $\mathcal{O}\left(1/\sqrt{\alpha_s}\right)$. By adopting a new power-counting with $\alpha_s^2 L_\perp \sim \mathcal{O}\left(1\right)$ and setting $\mu \sim \max\left[q_T, q_*\right]$, the terms of the CA lead to finite, resummed results, independent of the restriction $q_T > q_*$ or even $q_T > \Lambda_{NP}$. Thus the CA leads to infrared safety, in the sense that it gives the possibility to calculate the intercept at $q_T = 0$.

4 Conclusion and Outlook

The transverse-momentum distribution of Drell-Yan like processes is one of the most basic observables at hadron colliders. It nevertheless manifests a number of remarkable properties at low transverse-momentum. Our approach using SCET to factorize the differential cross section and resum large logarithms via RG-techniques, leads for the first time to an analytical result in momentum space, free of unphysical Landau-pole singularities. The CA creates a new scale q_*, which protects the cross section at vanishing transverse-momentum from non-perturbative effects, so it leads to infrared safety.

Numerical comparisons of our predictions with data on the transverse-momentum distribution in Z-boson production at the Tevatron and LHC are given in Fig. 1. They include the matching to NLO fixed-order calculations and the influence of long-distance effects, which are suppressed by q_*. The hard function is resummed using the π^2-resummation. The error bands are calculated by varying μ by a factor of two. All effects are discussed in detail in [2].

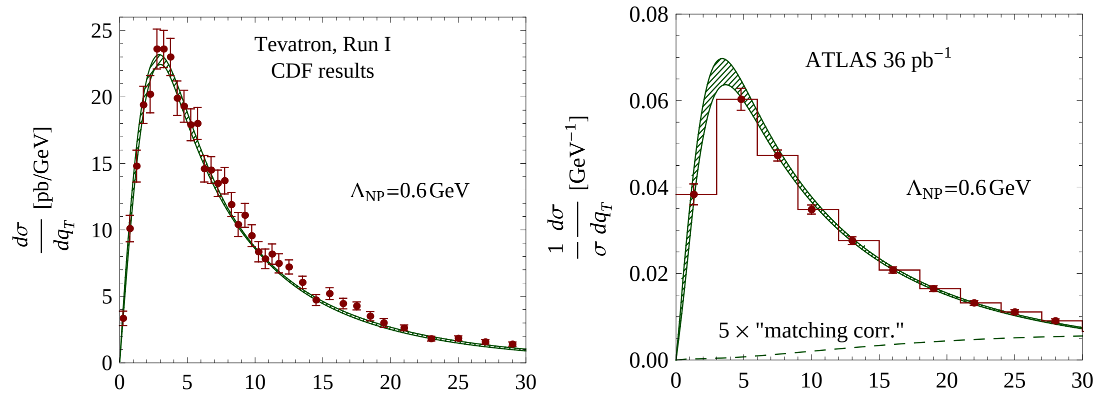

Figure 1: Transverse-momentum distribution compared with Tevatron Run I data from CDF [6] and LHC data from ATLAS [7].

Work in progress is the implementation of the lepton-tensor, in order to regard the experimental cuts and the extension to the Higgs-boson. In addition we need to match our resummed cross section to NNLO fixed-order results to extend our prediction to higher q_T. The next milestone in future will be to match the generalized PDFs at two-loop order, to improve our accuracy to the actual level of fixed-order calculations.

References

[1] T. Becher and M. Neubert. Eur.Phys.J. **C71** (2011) 1665, `arXiv:1007.4005 [hep-ph]`.

[2] T. Becher, M. Neubert, and D. Wilhelm. JHEP **1202** (2012) 124, `arXiv:1109.6027 [hep-ph]`.

[3] J. C. Collins, D. E. Soper, and G. F. Sterman. Nucl.Phys. **B250** (1985) 199.

[4] V. Sudakov. Sov.Phys.JETP **3** (1956) 65–71.

[5] T. Becher and G. Bell. `arXiv:1112.3907 [hep-ph]`.

[6] T. Affolder *et al.* Phys.Rev.Lett. **84** (2000) 845–850, `arXiv:hep-ex/0001021 [hep-ex]`.

[7] G. Aad *et al.* Phys.Lett. **B705** (2011) 415–434, `arXiv:1107.2381 [hep-ex]`.